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The effect of high magnetic field on the scattering of electron with atomic hydrogen

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Abstract : Investigation has been carried out to show the effect of high magnetic field over the scattering parameters for electron-hydrogen scattering. The magnitudes of the magnetic field vary from 0.2 to 5 a.u. Three first-order models have been employed to study the system. These models reveal the relative effect of high magnetic field on the projectile and the target both separately and simultaneously. The scattering cross sections are compared amongst themselves and also with the corresponding field-free results. It has been found that the effect of field distortion of the target is negligible on the scattering parameters. But the dressing of the incoming electron for the magnetic field bring significant qualitative change.

Keywords Electron, hydrogen, high magnetic field, scattering cross section

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1. Introduction

It is well known that the external field can change the flavour of basic physics. The Stark effect and the Zeeman effect are two famous examples. In particular, magnetic field has been employed to study various aspects of physical system. While exploring the various properties of magnetized plasma [1] or during the study of energy spectrum of hydrogen like excitons [2], existence of magnetic field alters the situation dramatically. Moreover, magnetic cooling, magnetic bottling, and magnetic trapping are frequently used in Bose Einstein Condensation as well as in the antiparticle physics [3,4]. The large numbers of theoretical investigations on hydrogen atom wave functions in presence of high magnetic field have opened up a new domain of quantum chaos [5,6]. Therefore, the scattering process in presence of external magnetic field can provide a large number of information about the structure and dynamics of the different atomic systems. Moreover, this kind of study has important application in revealing various properties of astrophysical system, *e.g.*, surface property of neutron star.

Motivated by the above facts, we plan to study the simplest scattering system *i.e.* scattering of an electron by a hydrogen atom in a strong magnetic field. This problem is much more complicated from the corresponding field-free case. First of all, the incident electron in presence of high magnetic field moves one dimensionally along the direction of the magnetic field. The electron looses its plane wave character and moves in cylindrical wave fashion, characterized by Landau levels having two quantum numbers n and s. To get the wave function of an electron in a particular n-th state, an infinite sum over the other quantum number s, has to be performed. Secondly, the exact wave function of the Hydrogen atom in presence of magnetic field is not known and is much more different from the corresponding field-free counterpart. An *ab initio* calculation for this system is rather difficult to perform.

Here, we investigate electron-hydrogen atom scattering in the presence of a strong magnetic field using first-order theory. We are interested to judge the qualitative change of the scattering parameters when the magnetic field is on, with respect to the field-free electron-hydrogen scattering. We take the field-free scattering parameters as a base line and employ three different models. In model (a), we assume the modified wave function of the incident electron in the magnetic field while hydrogen atom remain unperturbed during the interaction time. In the second model (b), we take one of the most accurate available variational wave function of the atomic hydrogen under the magnetic field [7], and the incident electron is considered to be free from the external magnetic field. To take the necessary account of the effect of the dressing of both colliding systems, we introduce the third picture, model (c) where the electron's wave function as well as the hydrogen's wave function are dressed by the magnetic field. These three models will help us to reveal the relative effect of strong magnetic field in the electron-atom scattering. To the best of our knowledge, this is the first quantum mechanical attempt to explore the magnetic-field-assisted electron-atom scattering. Previously, potential scattering of electron in the back ground of strong magnetic field has been studied by several workers [8-12].

2. Theory

We briefly describe our theoretical model in this section. Atomic units are used through out the paper. We take the magnetic field along z axis. The Schrödinger wave equation in presence of magnetic field can be written as

$$(H_{\rm int} + H_1 + H_2)\psi = E\psi , \tag{1}$$

where $H_{\rm int}$ is the interaction hamiltonian between electron and hydrogen ; H_1 and H_2 are the hamiltonian of free electron and the electron of the hydrogen atom in presence of the magnetic field, respectively.

$$H_1 = 1/2(\boldsymbol{P}_e - \boldsymbol{A})^2 \tag{2}$$

and

$$H_2 = 1/2(\boldsymbol{P}_H - \boldsymbol{A})^2 - 1/r_2 , \tag{3}$$

where $\boldsymbol{P}_e$ and $\boldsymbol{P}_H$ is the momentum of the free electron and the bounded electron, respectively and $\boldsymbol{A}$ is the vector potential representing the field and is given by

$$\boldsymbol{A} = \left(0.0, (1/2)H\ r \sin\theta\right). \tag{4}$$

The total wave function of the system is expressed as

$$\Psi = U(\boldsymbol{r}_1)\ \Omega(\boldsymbol{r}_2) , \tag{5}$$

where $U(\boldsymbol{r}_1)$ and $\Omega(\boldsymbol{r}_2)$ are the wave functions of the incoming electron and bounded electron, respectively. When electron is dressed by the magnetic field, the wave function $U(\boldsymbol{r}_1)$ satisfies the equation

$$H_1 U(\boldsymbol{r}_1) = \varepsilon\ U(\boldsymbol{r}_1) \tag{6}$$

with energy eigen value ε given by

$$\varepsilon = \frac{K^2}{2} + H\left(n + \frac{1}{2}\right). \tag{7}$$

Here, K is the one-dimensional momentum of the electron along the direction of the magnetic field. The magnetic field is given in atomic unit (2.35×10^5 tesla equals to 1 a.u.). However, in cylindrical coordinate, $U(\boldsymbol{r}_1)$ is nothing but well known Landau level and is given by

$$U_{nsk} = e^{ikz}\Phi_{ns}(\rho,\theta) , \tag{8}$$

where $\Phi_{ns}(\rho,\theta)$ is the cylindrical wave function (here we adopt the convention of Ventura[8], for other choice see [9]). k is the wave number along Z axis, n is the principle quantum no. and s is another quantum no. which for a particular n, runs from zero to infinity. The first order transition amplitude is given by

$$B_1 = \left\langle U_f(r_1)\Omega_f(r_2)\middle|V_2\middle|U_i(r_1)\Omega_i(r_2)\right\rangle \tag{9}$$

Here, V is the interaction potential between target and projectile. The subscript i and f refer to the initial and final channel wave functions, respectively.

In model (a) and (c), $U(\boldsymbol{r}_1)$ is represented by eq. (8) where as in model (b), it is plane wave. For model (a) and (c), B_1 involves sum over s_i and s_f. We use ordinary field-free hydrogen wave function in model (a), whereas in model (b), we take the magnetized hydrogen atom wave function due to Gallas [7] In parabolic coordinates, the wave function is given by :

$$\Omega(\xi,\eta) = N\pi^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(a\xi + b\eta + c\xi\eta)\right. \tag{10}$$

where N is the magnetic field-dependent normalization constant and a, b, c are the variational parameters. For the ground state, the identity $a = b$ always holds. This variational wave function is reliable and valid for a large span of magnetic field. Finally in model (c), as wave functions of both the target and the projectile are dressed, we have used simple wave function due to Gallas [13]. In this wave function, the radial part of the magnetized hydrogen atom is taken as

$$R_{n,l,m}(r) = \exp[-\beta r] , \tag{11}$$

whereas the angular part of the wave function remains as usual This wave function is very similar to field-free hydrogen atom However, the range parameter β of the radial part is used as a variational parameter. This basis set is used to obtain the energy eigen-value of the hamiltonian described in eq. (3). The energy minimization procedure is now applied to get the value of β and the optimum value of energy.

Using that sorts of wave function, we obtain the first Born cross section for all the models. To evaluate the first-order transition amplitude (B_1), we have used Gauss-Legendre quadrature technique to get the numerical results. The convergence of the results are tested. Moreover, by augmenting our model (a) and model (c), we can reproduce the results of Ventura [8] and in the zero field situation, our model (b) gives the field-free scattering results. For model (a) and (c), the definition of the cross section of a particular transition from n_i to n_f states is given by

$$\sigma = \left.\frac{2\pi}{H}\right)_{k_i|k_f} (B_1)^2. \tag{12}$$

In the model (b), the definition of the cross section is same as the field-free case.

3. Results and discussion

We consider here the direct elastic scattering of an electron off an atomic Hydrogen in presence of high magnetic field. In model (a) and (c) where we have taken the dressing of the incident electron in magnetic field, the scattering process is one-dimensional; whereas in the other two models the scattering is three-dimensional. In model (a) and (c), the forward (backward) scattering corresponds to the scattering of the electron along (against) the direction of the applied field. In models (b) and the field-free case, by forward cross section we mean the probability of the electron to scatter to the forward hemisphere ($0 \leq \theta \leq \pi/2$) and backward cross section corresponds to the scattering to the backward hemisphere. The forward, backward and the total scattering cross sections are evaluated and compared using different models. Here, we report the variation of the cross section with the energy of the incoming electron taking magnetic field as a parameter. The magnetic field values have been varied from 0.2 a.u. to 5 a.u. The magnetic field value which affects the scattering procedure appreciably is of present interest and we report these results.

At 0.2 a.u. magnetic field (not shown), the results of model (a) and model(c) coalesce up to the incident energy of nearly 1 a.u. Afterwards, marginal difference in results between the two appears, the results of model (a) being higher. Similarly, the results of model (b) and field-free results coincide with each other for whole the energy region. The same characteristic nature is found for the magnetic field value of 1 a.u. It implies that magnetic field value cannot influence the forward scattering cross section below the magnetic field value of 1 a.u. At 1 a.u. magnetic field, the behaviour of the forward scattering cross section are shown in the Figure 1. It is noticed from the figure that the results of four different models are well separated. On the other hand, high magnetic field ($H \geq 1$ a.u.) affects the elastic forward cross section appreciably which is evident from Figure 2. The results of all the models differ significantly at 5 a.u. of magnetic field for the whole energy range.

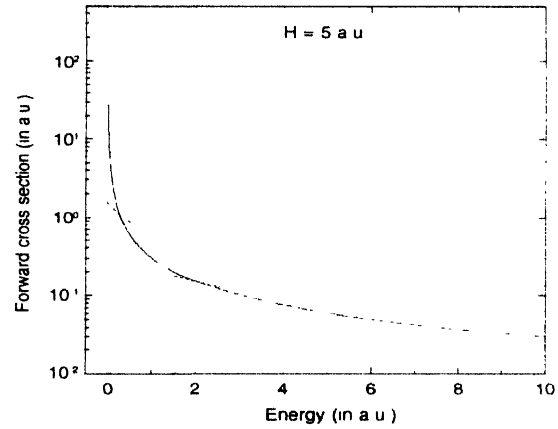

Figure 2. Forward cross section at H=5 a u Curves .... model(a) , —— model (b) ; ___ model(c) , -•-•- zero field result.

In the case of backward scattering up to 1 a.u. of magnetic field, the predictions of model (a) and model (c) coalesce each other as the model (b) does with the field-free model for the whole energy region (Figure 3). However, as in the case of forward scattering, all these results become separated at higher magnetic field (Figure 4). We conclude from Figures 3 and 4 that

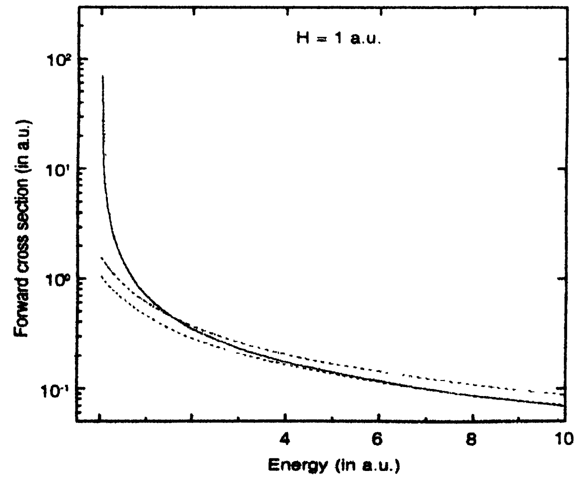

Figure 1. Forward cross section at H=1 a.u. Curves: .... model(a) ; model (b) ; ___ model(c) ; -•-•- zero field result.

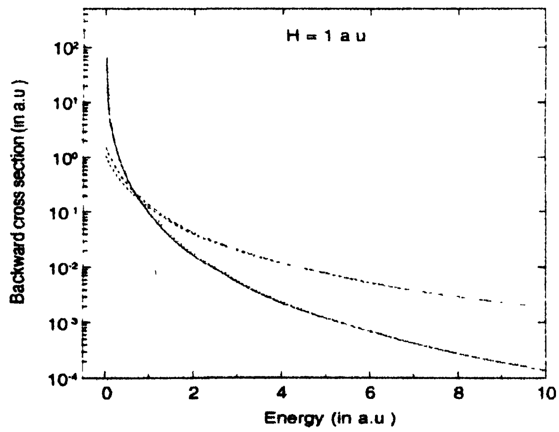

Figure 3. Backward cross section at H=1 a.u. Curves: .... model(a) model (b) ; ___ model(c) ; -•-•- zero field result.

the differences of the backward scattering cross sections of model (a) with model(c) and that of model (b) with the field-free case occur in the low energy regime. Also the results predicted by the one-dimensional models ((a) and (c)) and that predicted by three-dimensional models ((b) and field-free) become close to each other in the high energy region as magnetic field increases. These two features are different from the forward cross section behaviour. It is noticed from Figure 4 and also from Figures 2 and 3 that at high enrgy region, the backward cross sections of two different models are going to meet while the forward cross sections are well separated. Thus, we hasten to add that the distortion of the target due to the magnetic field is prominent in the forward channel than in the backward channel.

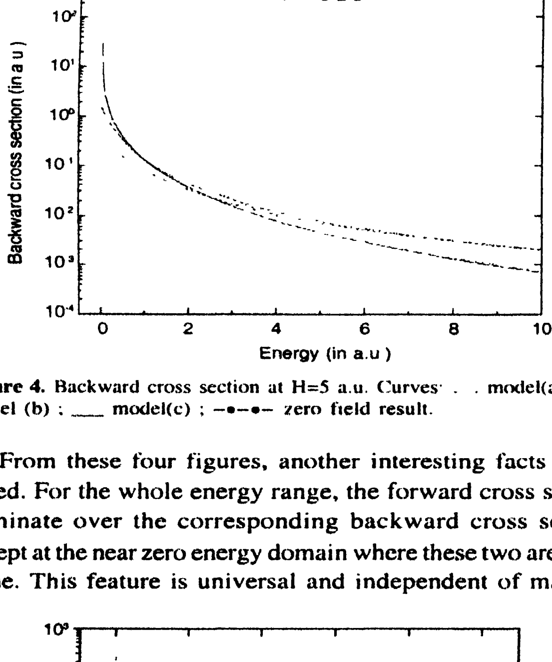

Figure 4. Backward cross section at H=5 a.u. Curves . . . model(a) ; —— model (b) ; ___ model(c) ; —•—•— zero field result.

From these four figures, another interesting facts can be noted. For the whole energy range, the forward cross sections dominate over the corresponding backward cross sections except at the near zero energy domain where these two are nearly same. This feature is universal and independent of magnetic

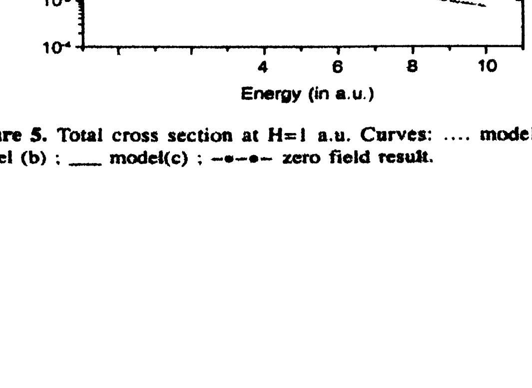

Figure 5. Total cross section at H=1 a.u. Curves: .... model(a) model (b) ; ___ model(c) ; —•—•— zero field result.

field and model. The behaviour of the total cross section (consists of forward and backward cross section) is shown in Figure 5. The qualitative feature of the total cross section of all the models are like the forward cross section as they are dominating.

4. Conclusion

We have investigated the elastic electron-hydrogen scattering at high magnetic field. The results are presented to show the qualitative difference from the field-free scattering parameter for a limited region of magnetic field strength. However, using the same methodology, one can extend it to the other field value of interest. The effect of high magnetic field are considered on the projectile and target, separately and simultaneously on both of them. Results are compared among them also with the field-free scattering parameter. In the models (a) and (c), the dressing of the incoming electron is taken. These two results change both qualitatively and quantitatively from the field-free results. Therefore, the dressing of the projectile is more important phenomenon. Also, in these two models, the near zero energy elastic forward and backward scattering cross sections diverge. Therefore, we hasten to add that this divergence is a feature of the electron's motion in magnetic field. On the other hand, in model (b), only the target is dressed and for all the concerning magnetic field, this results are similar to those of field-free results. Also below 1 a.u. of magnetic field, the forward as well as backward elastic cross section of model (a) and model (c) are almost identical. Therefore, the effect of target distortion due to magnetic field is nominal and not very significant below 1 a.u. magnetic field so far as the elastic scattering is concerned. The incoming electron being free, is largely affected by the applied magnetic field. In fact, the plane wave geometry is changed into cylindrical geometry and the three-dimensional motion is reduced to one dimension. This brings large change in scattering process. Hydrogen atom ground state on the other hand, being spherically symmetric, is least affected by the external magnetic field. Classically, it can be justified that the hydrogen atom is a diamagnetic substance [14] and so magnetic field could not bring large change in it. Difference of the qualitative behaviour of forward and backward cross section, shows that the effect of high magnetic field is significant in the forward channel. Due to the unavailability of any theoretical or experimental data, we could not judge the merits of the different models. Still we hope that our predictions could serve as a platform for an unexplored domain of quantum scattering. Also, it would be interesting to employ higher order approximation to study the problem.

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